

Solving Quadratics

(1.4)



What is a quadratic?

- Polynomial of 2nd degree

- Standard form: $ax^2 + bx + c = 0$ \swarrow 2 solutions!

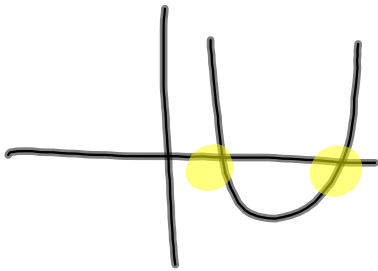
- graph is parabolic

To Solve Quadratics

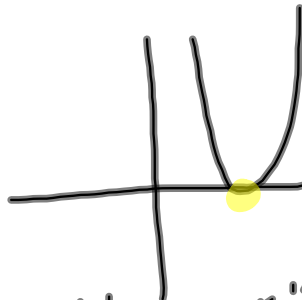
- Quadratic Formula
- Factoring
- Graphing
- Square rooting (if $b = 0$)
- Completing the square

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

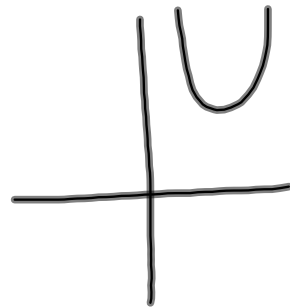
- Solution
- intercepts
- zero's



2 real sol.



"bouncer"
2 repeated sol.



2 imag. sol.

Discriminant:

$$b^2 - 4ac$$

$b^2 - 4ac$	positive	two solutions <i>✓ real</i>
$b^2 - 4ac$	perfect square zero	2 rational sol. <i>* factor nicely</i> one solution <i>repeated "bouncer"</i>
$b^2 - 4ac$	negative	no <i>real</i> solution 2 imaginary sol.

Solve by factoring.

$$4x^2 - 10x - 6 = 0$$

$$2(2x^2 - 5x - 3) = 0$$

$$(2x + 1)(x - 3) = 0$$

$$2x + 1 = 0 \quad x - 3 = 0$$

$$x = -\frac{1}{2} \quad x = 3$$

Zero product
property

$$x \cdot y = 0$$

$$x = 0 \text{ or } y = 0$$

discriminant

$$25 - 4(2)(-3)$$

$$49$$

$$x^2 + 8x = 4$$

$$9x^2 - 25 = 0$$

$$(3x-5)(3x+5)=0$$

$$x = \frac{5}{3} \quad x = -\frac{5}{3}$$

$$9x^2 = 25$$

$$x^2 = \frac{25}{9}$$

$$x = \pm \sqrt{\frac{25}{9}}$$

$$x = \pm \frac{5}{3}$$

$$(x - 3)^2 = 7$$

$$x - 3 = \pm\sqrt{7}$$

$$x = 3 \pm \sqrt{7}$$

Completing the square:

- force a quadratic to become a perfect square trinomial
- leading coefficient ~~must be~~ 1
(a = 1) *easier*

$$6x = 4 - x^2$$

$$x^2 + 6x - 4 = 0 \quad \leftarrow \text{Discriminant } 36 - 4(1)(-4)$$

$$x^2 + 6x + 9 = 4 + 9$$

$$(x + 3)^2 = 13$$

$$x + 3 = \pm\sqrt{13}$$

$$x = -3 \pm \sqrt{13}$$

$$3x^2 - 18x + 25 = 0$$

$$3x^2 - 18x = -25$$

$$3(x^2 - 6x + 9) = -25 + 27$$

$$3(x-3)^2 = 2$$

$$(x-3)^2 = \frac{2}{3}$$

$$x-3 = \pm\sqrt{\frac{2}{3}} \rightarrow x = 3 \pm \sqrt{\frac{2}{3}}$$

$$9x^2 - 12x = 14$$

$$\underbrace{9x^2 - 12x + 4}_{(3x-2)^2} = 14 + 4$$

$$(3x-2)^2 = 18$$

$$3x-2 = \pm\sqrt{18}$$

$$3x = 2 \pm 3\sqrt{2}$$

$$x = \frac{2 \pm 3\sqrt{2}}{3}$$

Solve using any method:

$$3x + 4 = 2x^2 - 7$$

$$-2x^2 + 3x + 11$$

$$x = \frac{-3 \pm \sqrt{97}}{-4}$$

Discriminant

$$9 - 4(-2)(11)$$

$$9 + 88$$

$$(97)$$

doesn't
factor
nicely!

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\frac{b}{a} \cdot \frac{1}{2} \left(\frac{b}{2a} \right)^2$$

$$\left(x + \frac{b}{2a} \right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$